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Domain walls in noncommutative field theories

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Abstract: We study kinks in a wide class of noncommutative (NC) field theories. We find rich structure of the static kinks in DBI type NC tachyon action for an unstable Dp-brane with general constant open string metric and NC parameter. Among which thick topological BPS NC kink and tensionless half-kink are particularly intriguing. Reproduction of the correct decent relation between Dp and D$(p-1)$ lets us interpret the obtained NC kinks as codimension-one D-brane and its composites. If we turn on DBI type NC U(1) gauge field on an unstable D2-brane, only constant field strength is allowed by gauge equation and NC Bianchi identity. Inclusion of the NC U(1) gauge field induces fundamental string charge localized on the codimension-one brane, which turns D$(p-1)$ into D$(p-1)$F1.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Tachyon Condensation.

Contents

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1. Introduction

Dynamics of an unstable D-brane or D-brane-anti-D-brane in string theory has attracted much attention [1]. Instability of such system is represented by tachyonic degree and its dynamics is depicted by condensation of the tachyon field and related phenomena. Recently real-time description of the homogeneous tachyon condensation has been made by finding rolling tachyon solutions in both boundary conformal field theory (BCFT) and effective field theory (EFT) [2, 3]. When the tachyon is condensed, the list of questions involves dynamical description of decaying processes including inhomogeneity [4] and the formation of final products after the unstable D-brane decays. For the latter question, either perturbative degrees namely closed string degrees are produced [5, 6] or nonperturbative lower-dimensional branes are formed, in which the important objects are stable codimension-one D-branes or their composites including confined fundamental string (F1) [5, 7, 8].

Tachyon dynamics has been studied by using both various string theoretic methods including BCFT, open string field theory, boundary string field theory (BSFT), $c = 1$ matrix model, and effective field theoretic languages including EFT, noncommutative field theory (NCFT), p-adic string theory. When constant Neveu-Schwarz (NS) antisymmetric two-form field exists, an effective description of string theory is to employ the NCFT [9]. Naturally dynamics of the unstable D-brane by using noncommutative (NC) tachyon condensation has also been an attractive topic from the beginning [10, 11, 12]. On the production of lower-dimensional branes, studies are mostly about codimension-two objects since

Gopakumar-Minwalla-Strominger (GMS) solitons [13] and their analogues [14] are basic solitonic objects for the analysis. These provide a good description of D$(p-2)$s from D$p\bar{\rm D}p$ system, however another representative example is production of stable D$(p-1)$-branes from an unstable Dp-brane with or without both NS two-form field and Dirac-Born-Infeld (DBI) type electromagnetism. Though there have been many studies on both NC rolling tachyon [15] and codimension-one branes in terms of EFT [5, 16, 7, 17, 18, 19] and BCFT [8], an NCFT study was recently performed for D0s from an unstable D1 without and with DBI electric field [20].

In this paper we study domain walls and tachyon kinks in general NCFT settings and a DBI type NCFT including NC tachyon field. In section 2, we consider NCFT actions of a real NC scalar field with usual kinetic and polynomial type potential terms, and find all possible NC domain wall configurations where many of them are given by exact solutions. Extensions to the action with nonpolynomial scalar potential and an EFT from p-adic string theory are also briefly discussed. In section 3, $(p+1)$-dimensional DBI type NC tachyon action in the background of general constant open string metric and NC parameter with runaway tachyon potential is taken into account. All possible regular static codimension-one solitons are obtained, which are classified by array of NC kink-antikink, single topological NC kink which are either BPS or non-BPS due to open string metric, tensionless NC half-kink, NC bounce, and hybrid of two NC half-kinks. Since the decent relation between tension of p-brane and $(p-1)$-brane is correctly reproduced, the obtained kinks are interpreted as D$(p-1)$-branes and their composites. In section 4, we turn on DBI type NC U(1) gauge field on a D2-brane in the background of open string metric and NC parameter. Though the analysis is complicated, every electric and magnetic component of the NC field strength tensor on the D2-brane is proved to be constant. The obtained NC tachyon kink configurations have the same functional shapes as those in section 3, but their interpretation leads to D$(p-1)$F1 and their composites, where F1s are localized in D$(p-1)$. We conclude in section 5 with brief summary and discussion.

2. Domain walls in NC scalar field theory

Let us begin this section with a well-known local field theory action of a real scalar field ϕ in (1+1)-dimensions

$$S_{\rm EFT2} = \int dtdx \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right], \tag{2.1}$$

where the scalar potential $V(\phi)$ has at least two stable vacua, say $\{\phi_i\,|\,dV/d\phi|_{\phi=\phi_i} = 0,$ $d^2V/d\phi^2|_{\phi=\phi_i} > 0,\ i = 1, 2, \ldots\}$, with vanishing cosmological constant $V(\phi_i) = 0$. It is also well-known that this model (2.1) supports static topological kinks (domain walls) as classical solutions. Here we briefly review how to obtain them.

Momentum density T_{01} vanishes for static configurations $\phi = \phi(x)$, and then pressure component T^{11} is rewritten as

$$T^{11} = \frac{1}{2}\left(\phi' - \sqrt{2V(\phi)}\right)\left(\phi' + \sqrt{2V(\phi)}\right), \tag{2.2}$$

where $\phi' = d\phi/dx$. A definition of Bogomol'nyi bound is given by vanishing pressure everywhere, which provides a first-order equation so-called BPS equation

$$\phi' = \pm\sqrt{2V(\phi)}\,. \tag{2.3}$$

For any static configuration satisfying the BPS equation (2.3) saturates BPS bound, i.e., energy has minimum value bounded by topological charge

$$\begin{align} H &= \int_{-\infty}^{\infty} dx\, T_{00} \tag{2.4}\\ &= \int_{-\infty}^{\infty} dx\, \left[\frac{1}{2}\phi'^2 + V(\phi)\right] \tag{2.5}\\ &= \int_{-\infty}^{\infty} dx\, \left[\frac{1}{2}\left(\phi' \mp \sqrt{2V(\phi)}\right)^2 \pm Q(\phi)'\right] \tag{2.6}\\ &\geq \pm\left[Q(\phi(\infty)) - Q(\phi(-\infty))\right], \tag{2.7} \end{align}$$

where $Q' = \sqrt{2V}\phi'$ and $\phi(\pm\infty)$ is one of ϕ_is. The equality in the last line (2.7) holds for any BPS object, and the difference corresponds to topological charge of a kink or an antikink in order of the signature in front of the formula (2.7). Because of this nature, any static solution satisfying first-order BPS equation (2.3) also satisfies second-order Euler-Lagrange equation automatically, which can be derived through small variation of the energy H (2.4).

Simplest model is given by ϕ^4-potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 \tag{2.8}$$

with degenerate superselective vacua of $i = 1, 2$ at $\phi_i = \pm v$. Substituting the scalar potential (2.8) into the BPS equation (2.3) with boundary conditions, we obtain exact kink (antikink) solution

$$\frac{\phi(x)}{v} = \pm\tanh\left[\frac{m_{\rm H}}{\sqrt{2}}(x - x_0)\right], \tag{2.9}$$

where $m_{\rm H} = \sqrt{\lambda}v$ is Higgs mass and an integration constant x_0 is a zero mode. The energy density T_{00} of a kink (antikink) is localized near its location x_0 with thickness $1/m_{\rm H}$

$$T_{00} = \frac{\lambda}{2}v^4 {\rm sech}^4\left[\frac{m_{\rm H}}{\sqrt{2}}(x - x_0)\right], \tag{2.10}$$

and its energy given by the topological charge (2.7) is

$$|Q(\phi(\infty)) - Q(\phi(-\infty))| = \frac{4}{3}\sqrt{\frac{\lambda}{2}}v^3\,. \tag{2.11}$$

Now let us consider corresponding NCFT of an NC scalar field $\hat{\phi}$. We easily read its action as

$$S_{\rm NC2} = \int dtdx\left[-\frac{1}{2}\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + \frac{v^2}{2}\hat{\phi}^2 - \frac{\lambda}{4}(\hat{\phi} * \hat{\phi})^2\right], \tag{2.12}$$

where star product between two NC fields, f and g, is defined by

$$f(x) * g(x) \equiv e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\zeta^\nu}} f(x+\xi)g(x+\zeta)|_{\xi=\zeta=0}\,. \tag{2.13}$$

In eq. (2.12) we have used the property that one star product can be replaced by an ordinary product in the action due to spacetime volume integration. In this simplest NCFT, star products in the quartic interaction term are the only difference between the ordinary action (2.1) and NCFT action (2.12). In general this noncommutativity, a way of introducing nonlocality, plays a key role for complex but attractive phenomena. At tree level codimension-two solitons are supported, so-called GMS solitons [13], and at loop level UV/IR-mixing was found [21]. We are interested in codimension-one extended objects given by classical soliton solutions of Euler-Lagrange equations. For any static object with single coordinate dependence, e.g., $\hat{\phi} = \hat{\phi}(x)$, every star product is simply replaced by ordinary product $\hat{\phi}(x) * \hat{\phi}(x) = \phi(x)^2$ and then every domain wall solution in ordinary scalar field theory is also a solution in the corresponding NCFT. Note that inclusion of flat transverse directions and higher polynomial terms do not change this property.

Let us look into a possibility to identify the obtained codimension-one BPS kink with a stable D$(p-1)$-brane. Since tension of the D$(p-1)$-brane is given by the energy (or equivalently the topological charge) of the kink (2.11), $\mathcal{T}_{p-1} = |Q(\phi(\infty)) - Q(\phi(-\infty))|$, and constant vacuum energy of the action (2.1) at the unstable vacuum defines tension of the unstable Dp-brane

$$\mathcal{T}_p = V(\phi = 0) = \frac{\lambda}{4}v^4\,, \tag{2.14}$$

one can read a sort of decent relation

$$\mathcal{T}_{p-1} = \frac{8\sqrt{2}}{3}\frac{1}{m_{\rm H}}\mathcal{T}_p\,. \tag{2.15}$$

Since the correct decent relation [22] is read as $\mathcal{T}_{p-1} = \pi R \mathcal{T}_p$ with $R = \sqrt{2}$ for superstring theory in our unit and the square of tachyonic pion mass m_ϕ^2 near the symmetric vacuum $\phi = 0$ is $m_\phi^2 = -m_{\rm H}^2 = -1/R^2$, the obtained decent relation (2.15) involves about 20% difference in its coefficient, i.e., $(\frac{8\sqrt{2}}{3})/\pi = 1.20$.

Let us conclude this section by introducing decent relations obtained from a few effective field theory models of tachyon dynamics for unstable bosonic D-branes. First example is $\ell = \infty$ field theory model with nonpolynomial potential

$$S = \mathcal{T}_p \int d^{p+1}x \left(-4e^{-\frac{\phi^2}{2}}\partial_\mu\phi\partial^\mu\phi - e^{-\frac{\phi^2}{2}}\right). \tag{2.16}$$

Note that this tachyon effective action was proposed in ref. [23] and can be understood as two-derivative truncation of BSFT [24]. Its NC version was also considered in ref. [12] with NC U(1) gauge field. By estimating mass of a tachyon kink connecting $T = -\infty$ and $T = \infty$, we obtain a decent relation $\mathcal{T}_{p-1} = (2/\sqrt{\pi})\pi R \mathcal{T}_p$ [25]. Since string theory result requires unity instead of $2/\sqrt{\pi}$, it involves 12.8% difference. Second example is d-dimensional p-adic string theory described by a nonlocal action

$$S = \frac{1}{G_{\rm s}^2}\frac{p^2}{p-1}\int d^dx \left(-\frac{1}{2}\phi p^{-\frac{1}{2}\partial^2}\phi + \frac{1}{p+1}\phi^{p+1}\right), \tag{2.17}$$

where G_s is open string coupling constant and p is an arbitrary prime number which is related with tension of the p-adic string $(2\pi\alpha_p^{'})^{-1} = (\ln p)^{-1}$. According to ref. [26], decent relation between q-brane and $(q-1)$-brane is read as

$$\mathcal{T}_{q-1} = \sqrt{\frac{2\pi p^{\frac{2p}{p-1}} \ln p}{p^2-1}}\, \mathcal{T}_q \overset{p\to\infty}{\longrightarrow} \sqrt{2\pi \ln p}\, \mathcal{T}_q = 2\pi\sqrt{\alpha_p^{'}}\, \mathcal{T}_q \tag{2.18}$$

which reproduces an exact decent relation in bosonic string theory for self-dual radius $R_{\rm sd} \overset{p\to\infty}{=} \sqrt{\alpha_p^{'}}$.

In this section we have shown that flat codimension-one static configurations represented by kinks in ordinary field theory of a real scalar field are identified by the NC kink solutions of NCFT of a real NC scalar field.

3. Tachyon kinks in NC tachyon field theory

Single flat unstable Dp-brane exists in string theory where p is odd for type-IIA string theory and even for IIB. Its instability is represented by a real tachyonic degree, and effective action of the tachyon field T is claimed to be DBI type [27]

$$S = -\frac{1}{g_{\rm s}(2\pi)^{\frac{p-1}{2}}} \int d^{p+1}x\, V(T)\sqrt{-\det(\eta_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T)}\,, \tag{3.1}$$

where flat Minkowski metric $\eta_{\mu\nu}$ is the closed string metric $g_{\mu\nu}$ of our interest, $B_{\mu\nu}$ is NS-NS two-form field assumed to be constant, and string coupling $g_{\rm s}$ is inversely proportional to tension of the Dp-brane as

$$\mathcal{T}_p = \frac{1}{g_{\rm s}(2\pi)^{\frac{p-1}{2}}}\,. \tag{3.2}$$

The tachyon potential $V(T)$ measures variable tension of the unstable D-brane, i.e., it can be any runaway potential connecting monotonically its maximum coinciding with the tension of Dp-brane and its minimum representing vanishing unstable Dp-brane. In our conventions, the potential obeys [22]

$$V(T=0) = 1 \qquad \text{and} \qquad V(T=\pm\infty) = 0\,. \tag{3.3}$$

When $B_{\mu\nu}$ is constant, it is formally equivalent with constant DBI type electromagnetic field strength $F_{\mu\nu}$ on the Dp-brane. Since all the tachyon kink configurations supported by the aforementioned DBI type effective tachyon action (3.1) are obtained in the constant DBI type electromagnetic field [7, 18], one can just read all the tachyon kinks in the background of constant NS-NS two-form field $B_{\mu\nu}$ by replacing each $F_{\mu\nu}$ component by that of $B_{\mu\nu}$.

If we are interested in the kink configurations of codimension-one $T = T(x)$ with constant background $B_{\mu\nu}$, the action (3.1) is then simplified as

$$S = -\mathcal{T}_p \int d^{p+1}x\, V(T)\sqrt{\beta_p - \alpha_{px} T'^2}\,, \tag{3.4}$$

where α_{px} is 11-component of the cofactor $C^{\mu\nu}$ of matrix $(X)_{\mu\nu} = \eta_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T$ and $\beta_p = -\det(\eta_{\mu\nu} + B_{\mu\nu})$. Rescaling the x-coordinate,

$$\tilde{x} = \frac{x}{\sqrt{|\alpha_{px}/\beta_p|}}\,, \tag{3.5}$$

the action (3.4) becomes

$$S = -\mathcal{T}_p\sqrt{|\alpha_{px}|}\int dt \int d\tilde{x} \int dy_1 \cdots dy_{p-1}\, V(T)\sqrt{\beta_p/|\beta_p| - (\alpha_{px}/|\alpha_{px}|)\tilde{T}'^2}\,, \tag{3.6}$$

where $\tilde{T}' = dT/d\tilde{x}$. Equation of motion is given by the conservation of x-component of pressure $T^{11} = \alpha_{px}(\mathcal{T}_p V/\sqrt{-\det(X)_{\mu\nu}})$, and it is rewritten by a first-order equation

$$\mathcal{E}_p = \frac{1}{2}T'^2 + U_p(T)\,, \tag{3.7}$$

where $\mathcal{E}_p = \beta_p/2\alpha_{px}$ and $U_p(T) = \alpha_{px}[\mathcal{T}_p V(T)]^2/2(T^{11})^2$. Since the equation of motion (3.7) has three parameters $(\beta_p, \alpha_{px}, T^{11})$, all the obtainable kink configurations are classified by those.

The signature of T^{11} is the same as that of α_{px} so signature of U_p is also equal to α_{px}. When α_{px} is negative, U_p becomes upside down and has minimum value $\alpha_{px}\mathcal{T}_p^2/2(T^{11})^2$ at $T = 0$ and maximum value 0 at $T = \pm\infty$ due to the runaway nature of the tachyon potential (3.3). Note that, for $p = 1$, α_{px} should always be nonnegative. As β_p decreases, we have (i) constant solution $T = 0$ for $\beta_p = \alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2$, (ii) oscillating solution between $-\pi R\sqrt{-\alpha_{px}/\beta_p}$ and $\pi R\sqrt{-\alpha_{px}/\beta_p}$ for $0 < \beta_p < \alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2$, (iii) monotonic increasing (decreasing) solution connecting $T = -\infty$ and $T = \infty$ with $\tilde{T}'(\pm\infty) = 0$ for $\beta_p = 0$, and (iv) another monotonic increasing (decreasing) solution connecting $T = -\infty$, and $T = \infty$ with $\tilde{T}'(\pm\infty) \neq 0$ for $\beta_p < 0$. When α_{px} is positive, U_p has maximum value $\alpha_{px}\mathcal{T}_p^2/2(T^{11})^2$ at $T = 0$ and minimum value 0 at $T = \pm\infty$. As β_p increases, we have (v) constant solution $T = \pm\infty$ for $\beta_p = 0$, (vi) convex-down (-up) solution connecting its minimum (maximum) and positive (negative) infinity for $0 < \beta_p < \alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2$, (vii) monotonic increasing (decreasing) solution connecting $T = 0$ and $T = \infty$ $(-\infty)$ with (viii) constant $T = 0$ solution for $\beta_p = \alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2$, and (ix) another monotonic increasing (decreasing) solution connecting $T = -\infty$ (∞) and $T = \infty$ $(-\infty)$. Except unstable and stable vacuum solutions at $T = 0$ and $T = \pm\infty$, the obtained solutions are interpreted as array of kink-antikink for (ii), topological kinks for (iii) and (iv), bounce for (vi), half-kink for (vii), and hybrid of two half-kinks for (ix). When α_{px}=0, there exists no nontrivial solution.

Here let us assume a specific tachyon potential satisfying eq. (3.3) [28, 29, 30]

$$V(T) = \frac{1}{\cosh\left(\frac{T}{R}\right)}\,, \tag{3.8}$$

where R is $\sqrt{2}$ for the non-BPS D-brane in the superstring and 2 for the bosonic string. This form of the potential has been derived in open string theory by taking into account the fluctuations around $\frac{1}{2}$S-brane configuration with the higher derivatives neglected, i.e., $\partial^2 T = \partial^3 T = \cdots = 0$ [31].

For the $1/\cosh$-potential (3.8) the tachyon equation (3.7) is solved and we obtain exact kink solutions [7, 18],

$$\sinh\left(\frac{T(x)}{R}\right) = \begin{cases} \sqrt{u^2+1}\sinh(x/\zeta) & \text{for } \alpha_{px}<0,\ \beta_p<0 & \text{(iv)} \\ ux/\zeta & \text{for } \alpha_{px}<0,\ \beta_p=0 & \text{(iii)} \\ \sqrt{u^2-1}\sin(x/\zeta) & \text{for } \alpha_{px}<0,\ 0<\beta_p<\alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2 & \text{(ii)} \\ \sqrt{u^2-1}\cosh(x/\zeta) & \text{for } \alpha_{px}>0,\ 0<\beta_p<\alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2 & \text{(vi)} \\ \exp(x/\zeta) & \text{for } \alpha_{px}>0,\ \beta_p=\alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2 & \text{(vii)} \\ \sqrt{1-u^2}\sinh(x/\zeta) & \text{for } \alpha_{px}>0,\ \beta_p>\alpha_{px}^2\mathcal{T}_p^2/(T^{11})^2 & \text{(ix)} \end{cases}, \tag{3.9}$$

where the scales in the solutions are

$$u^2 = \left|\frac{\mathcal{T}_p^2\alpha_{px}^2}{\beta_p(T^{11})^2}\right|, \qquad \zeta = R\sqrt{\left|\frac{\alpha_{px}}{\beta_p}\right|}. \tag{3.10}$$

Computation of tension and F1 charge leads to interpretation of D$(p-1)$-brane (and D$(p-1)$F1) for single topological kink in (iii) and (iv), and array of D$(p-1)\bar{\rm D}(p-1)$ (and D$(p-1)$F1-$\bar{\rm D}(p-1)$F1) for array of kink-antikink in (ii), where F1 is confined on D$(p-1)$ in the form of string fluid. For (ii) and (iii), BCFT calculation confirms this interpretation [8]. The functional form of last three solutions in eq. (3.9) coincide with exact rolling tachyon solutions [29, 32]. This can be explained by the signatures in the rescaled action (3.6), i.e., $\beta_p/|\beta_p| = 1$ and $\alpha_{px}/|\alpha_{px}| = 1$ for those solutions, which coincide with those in the action for the rolling tachyons.

In relation with BPS nature, single topological kink (iii) saturates BPS type bound with thickness for $T^{11} \neq 0$ but each kink (antikink) in the array (iv) becomes a BPS object only when its thickness vanishes ($T^{11} \to 0$). In what follows let us study codimension-one tachyon solitons in NC DBI type action.

From the action (3.1), the flat closed string metric $\eta_{\mu\nu}$ and the NS-NS two-form field $B_{\mu\nu}$ are replaced by open string metric $G_{\mu\nu}$ and noncommutativity parameter $\theta^{\mu\nu}$

$$G_{\mu\nu} = \eta_{\mu\nu} - (B\eta^{-1}B)_{\mu\nu}\,, \tag{3.11}$$

$$\theta^{\mu\nu} = -\left(\frac{1}{\eta+B}B\frac{1}{\eta-B}\right)^{\mu\nu}, \tag{3.12}$$

where $-\det G_{\mu\nu} \geq 0$. Once the tachyon field T is turned on in the effective action (3.1), corresponding NC tachyon $\hat{T}$ should be introduced in the NC action. In ref. [20], a candidate was proposed

$$\hat{S} = -\frac{\hat{\mathcal{T}}_p}{2}\int d^{p+1}x\left[\hat{V}(\hat{T}) * \sqrt{-\det{}_*\left[G_{\mu\nu} + \frac{1}{2}(\partial_\mu\hat{T} * \partial_\nu\hat{T} + \partial_\nu\hat{T} * \partial_\mu\hat{T})\right]} + (\surd \leftrightarrow \hat{V})\right], \tag{3.13}$$

where comparison of two actions (3.1) and (3.13) with turning off both tachyon fields leads to a relation between the tensions

$$\hat{\mathcal{T}}_p \equiv \frac{1}{G_{\rm s}(2\pi)^{\frac{p-1}{2}}} = (-G)^{-1/4}\mathcal{T}_p = \frac{\mathcal{T}_p}{\sqrt{\det(1+\eta^{-1}B)}}\,, \tag{3.14}$$

where $G = \det(G_{\mu\nu})$. The determinant with subscript $*$ in the action (3.13) denotes

$$\det{}_* \hat{X}_{\mu\nu} = \frac{1}{(p+1)!}\epsilon_{\mu_1\mu_2\cdots\mu_{p+1}}\epsilon_{\nu_1\nu_2\cdots\nu_{p+1}}\hat{X}_{\mu_1\nu_1} * \hat{X}_{\mu_2\nu_2} * \cdots * \hat{X}_{\mu_{p+1}\nu_{p+1}} . \tag{3.15}$$

The NC tachyon potential $\hat{V}(\hat{T})$ imitates properties of that in EFT, so that in general $\hat{V}(\hat{T})$ can have any functional form satisfying $\hat{V}(\hat{T}=0)=1$ and $\hat{V}(\hat{T}=\pm\infty)=0$ as given in eq. (3.3) and the specific form of our interest is again 1/cosh form:

$$\begin{aligned}\hat{V}(\hat{T}) &\equiv 1 - \frac{1}{2}\frac{\hat{T}}{R} * \frac{\hat{T}}{R} + \frac{5}{24}\frac{\hat{T}}{R} * \frac{\hat{T}}{R} * \frac{\hat{T}}{R} * \frac{\hat{T}}{R} + \cdots \\ &= 1 + \sum_{k=1}^{\infty}\frac{E_{2k}}{(2k)!}\left[\left(\frac{\hat{T}}{R}\right)^{2k}\right]_* = \left[\frac{1}{\cosh(\hat{T}/R)}\right]_* ,\end{aligned} \tag{3.16}$$

where $[\quad]_*$ stands for

$$\begin{aligned}\left[\hat{A}_1\hat{A}_2\cdots\hat{A}_n\right]_* = \frac{1}{n!}\Big(&\hat{A}_1 * \hat{A}_2 * \cdots * \hat{A}_n + \hat{A}_1 * \hat{A}_3 * \cdots * \hat{A}_n + \\ &+ \cdots \text{(all possible permutations)}\Big) .\end{aligned} \tag{3.17}$$

Since the objects of our interest are flat static kink solutions given with a function of a coordinate x as

$$\hat{T} = \hat{T}(x) \tag{3.18}$$

in the flat background with constant $G_{\mu\nu}$ and $\theta^{\mu\nu}$, every star product of two NC fields is always replaced by ordinary multiplication as

$$\hat{T} * \hat{T} = \hat{T}^2 , \qquad (\partial_\mu\hat{T}) * \hat{T} = \delta_{\mu 1}\hat{T}'\hat{T} , \qquad \partial_\mu\hat{T} * \partial_\nu\hat{T} = \delta_{\mu 1}\delta_{\nu 1}\hat{T}'^2 . \tag{3.19}$$

Substituting the formulae (3.19) into the action (3.13), we obtain again a simplified action

$$\hat{S} = -\hat{\mathcal{T}}_p \int d^{p+1}x\, \hat{V}(\hat{T})\sqrt{\hat{\beta}_p - \hat{\alpha}_{px}\hat{T}'^2} , \tag{3.20}$$

where $\hat{\beta}_p = -\det(G_{\mu\nu}) \geq 0$ and $\hat{\alpha}_{px}$ is 11-component of the cofactor $\hat{C}^{\mu\nu}$ of matrix $(\hat{X})_{\mu\nu} = G_{\mu\nu} + \partial_\mu\hat{T}\partial_\nu\hat{T}$. When $\hat{\beta}_p$ is not zero, we can rewrite the action (3.20) as

$$\hat{S} = -\hat{\mathcal{T}}_p\sqrt{|\hat{\alpha}_{px}|}\int dt \int d\tilde{x}\int dy_1\cdots dy_{p-1}\,\hat{V}(\hat{T})\sqrt{1 - \frac{\hat{\alpha}_{px}}{|\hat{\alpha}_{px}|}\left(\frac{d\hat{T}}{d\tilde{x}}\right)^2} , \tag{3.21}$$

where

$$\tilde{x} \equiv \sqrt{\frac{\hat{\beta}_p}{|\hat{\alpha}_{px}|}}\, x .$$

Note that the action (3.20) is formally the same as that in eq. (3.4) with replacing the quantities without hat by those with hat, e.g.,

$$\mathcal{T}_p \leftrightarrow \hat{\mathcal{T}}_p , \quad T \leftrightarrow \hat{T} , \quad V \leftrightarrow \hat{V} , \quad \beta_p \leftrightarrow \hat{\beta}_p , \quad \alpha_{px} \leftrightarrow \hat{\alpha}_{px} . \tag{3.22}$$

Therefore, we can automatically read the equation of motion for static codimension-one objects as

$$\hat{\mathcal{E}}_p = \frac{1}{2}\hat{T}'^2 + \hat{U}_p(\hat{T})\,, \tag{3.23}$$

where

$$\hat{\mathcal{E}}_p = \frac{\hat{\beta}_p}{2\hat{\alpha}_{px}}\,, \qquad \hat{U}_p(\hat{T}) = \frac{\hat{\alpha}_{px}}{2\hat{\beta}_p(\hat{T}^{11})^2}[\hat{\mathcal{T}}_p\hat{V}(\hat{T})]^2\,. \tag{3.24}$$

Here we must caution that $(T^{11})^2$ is replaced by $(\hat{T}^{11})^2\hat{\beta}_p$. This is just because that the NC energy-momentum tensor in NCFT is defined under the open string metric $G_{\mu\nu}$:

$$\begin{aligned}\hat{T}^{\mu\nu} &\equiv \frac{2}{\sqrt{-G}}\frac{\delta\hat{S}}{\delta G_{\mu\nu}} &(3.25)\\ &= \frac{\hat{\mathcal{T}}_p\hat{V}\hat{C}^{\mu\nu}_{\mathrm{S}}}{\sqrt{-G}\sqrt{-\hat{X}}} &(3.26)\\ &= \frac{\hat{\mathcal{T}}_p\hat{V}}{\sqrt{-\hat{X}}\sqrt{-G}}\left(\hat{X}G^{\mu\nu} - GG^{\mu\lambda}\partial_\lambda\hat{T}G^{\nu\kappa}\partial_\kappa\hat{T}\right), &(3.27)\end{aligned}$$

where $\hat{X} \equiv \det\hat{X}_{\mu\nu}$, $\hat{C}^{\mu\nu}_{\mathrm{S}}$ is symmetric part of the cofactor $\hat{C}^{\mu\nu}$, and x-component of conservation of NC energy-momentum tensor

$$\partial_\mu\hat{T}^{\mu\nu} = 0 \tag{3.28}$$

forces $\hat{T}^{11}$ to be a constant.

In order to compare classical NC tachyon kink solutions which will be obtained in the subsections 3.1–3.6 and those in BCFT, we should compare components of energy-momentum tensor and string current density, i.e., they are $T_{\mu\nu}$ vs. $T^{\mathrm{BCFT}}_{\mu\nu}$ and $J_{\mu\nu}$ vs. $J^{\mathrm{BCFT}}_{\mu\nu}$. To be specific, the energy-momentum tensor $T_{\mu\nu}$ and the string current density $J_{\mu\nu}$ are given by responses to small fluctuations of the background closed string metric $g_{\mu\nu}$ and the anti-symmetric tensor field $B_{\mu\nu}$:

$$\begin{aligned}T_{\mu\nu} &\equiv -\frac{2}{\sqrt{-g}}\frac{\delta\hat{S}}{\delta g^{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}} &(3.29)\\ &= \frac{\mathcal{T}_pV}{(gG)^{1/4}\sqrt{-\hat{X}}}\left[\frac{1}{2}\hat{X}g_{\mu\nu} + (g_{\mu\lambda}g_{\nu\kappa} - B_{\mu\lambda}B_{\nu\kappa})\left(\frac{1}{2}\hat{X}G^{\lambda\kappa} - G\partial^\lambda\hat{T}\partial^\kappa\hat{T}\right)\right]\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}\,,\\ J^{\mu\nu} &\equiv \frac{2}{\sqrt{-g}}\frac{\delta\hat{S}}{\delta B_{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}} &(3.30)\\ &= -\frac{\mathcal{T}_pV}{(gG)^{1/4}\sqrt{-\hat{X}}}\left[\left\{-\frac{1}{2}\hat{X}G^{\mu\lambda}B_{\lambda\kappa}g^{\kappa\nu} + G\partial^\mu\hat{T}\partial^\lambda\hat{T}B_{\lambda\kappa}g^{\kappa\nu}\right\} - \{\mu\leftrightarrow\nu\}\right]\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}\,,\end{aligned}$$

For the derivation of eqs. (3.29)–(3.30), we used

$$\begin{aligned}\delta G_{\mu\nu} &= -(g_{\mu\lambda}g_{\nu\kappa} - B_{\mu\lambda}B_{\nu\kappa})\,\delta g^{\lambda\kappa} -\\ &\quad -\frac{1}{2}\left(\delta_\mu{}^\lambda g^{\kappa\rho}B_{\rho\nu} + \delta_\nu{}^\lambda g^{\kappa\rho}B_{\rho\mu} - (\lambda\leftrightarrow\kappa)\right)\delta B_{\lambda\kappa}\,,\end{aligned}$$

$\hat{U}_p(0)$	$\hat{\mathcal{E}}_p$	Type of NC kinks	Exact solutions
$\hat{U}_p(0) < 0$	$\hat{\mathcal{E}}_p > 0$	Topological NC kink	(iv)
	$\hat{\mathcal{E}}_p = 0$	Thick topological BPS NC kink	(iii)
	$\hat{U}_p(0) < \hat{\mathcal{E}}_p < 0$	Array of NC kink-antikink	(ii)
$\hat{U}_p(0) > 0$	$0 < \hat{\mathcal{E}}_p < \hat{U}_p(0)$	NC bounce	(vi)
	$\hat{\mathcal{E}}_p = \hat{U}_p(0)$	NC half-kink	(vii)
	$\hat{\mathcal{E}}_p > \hat{U}_p(0)$	Hybrid of two NC half-kinks	(ix)

Table 1: List of regular static NC kink configurations. Functional form of exact NC kink solutions in the fourth column are read from eq. (3.9) under the replacement (3.22).

and note that eqs. (3.29)–(3.30) coincide with those from commutative tachyon effective action [7, 18].

The equation (3.23) derived from the simplified action (3.20) is consistent with the NC tachyon equation derived directly from the original NC action (3.13). First observation is made by comparing the expressions in eq. (3.23) with those in eq. (3.7): One is

$$\hat{\mathcal{E}}_p = \frac{\hat{\beta}_p}{2\hat{\alpha}_{px}} = \frac{\beta_p}{2\alpha_{px}} = \mathcal{E}_p \tag{3.31}$$

with the help of eq. (3.11), and, the other is, when the NC tachyon field $\hat{T}(x)$ is identified by the tachyon field $T(x)$ of EFT which results in $\hat{V}(\hat{T}) = V(T)$,

$$\hat{U}_p = U_p \tag{3.32}$$

with the help of eq. (3.14). From eq. (3.26), we read the relation between $\hat{T}^{11}$ and T^{11}

$$\sqrt{-G}\,\hat{T}^{11} = T^{11}\,, \tag{3.33}$$

where we have used $\hat{C}^{11} = \hat{C}^{11}|_{\hat{T}=0}$ and $\sqrt{-X/-\hat{X}} = \sqrt{-\det(\eta+B)/-\det(G_{\mu\nu})}$. It means that every kink solution obtained from the EFT given in the first half of this section can also be a solution of the corresponding NCFT of the NC tachyon (3.23) as far as the aforementioned relations between eq. (3.31) and eq. (3.32) are satisfied.

We do not need to recapitulate presenting species of the solutions here due to the aforementioned analysis between eq. (3.7) and eq. (3.9). Instead we summarize the viable NC kink solutions in table 1, specified by the value and signature of $\hat{\mathcal{E}}_p$ and $\hat{U}_p(\hat{T}=0)$. In particular, $\hat{U}_p(\hat{T}=0)$ should be nonpositive for $p=1$. Therefore, only the first three types of NC kinks in table 1 are obtained from the unstable D1-brane [20]. On the other hand, $\hat{U}_p(\hat{T}=0)$ can be arbitrary for $p \geq 2$ so that all the six types of NC kinks in table 1 can be obtained.

If we naively look at the expressions of $\hat{\mathcal{E}}_p$ and $\hat{U}_p$ in eq. (3.24), signature of $\hat{\mathcal{E}}_p$ and that of $\hat{U}_p$ should be the same and equal to that of $\hat{\alpha}_{px}$ due to nonnegativity of $\hat{\beta}_p$, i.e., $\hat{\beta}_p = -G = -\det(G_{\mu\nu}) = [-\det(\eta+B)]^2 \geq 0$. If it is indeed the case, then the topological NC kink (iv) in the second low of table 1 cannot be supported. This observation was made under the assumption that all the squared quantities in eq. (3.24) are nonnegative. This needs not to be true for $\hat{\mathcal{T}}_p$ introduced in eq. (3.14). The condition we have is $-G \geq 0$, so imaginary $(-G)^{-1/4} = 1/\sqrt{\det(1+\eta^{-1}B)}$ is not excluded. For this case, the tension $\mathcal{T}_p$ is real positive and thereby $\hat{\mathcal{T}}_p$ (3.14) is imaginary. This phenomenon corresponds to electromagnetic fields larger than critical value with $-\det(\eta + B) < 0$ in the EFT, and this situation is not allowed without the NC tachyon ($\hat{T} = 0$) since the NC action (3.20) becomes imaginary. Once we turn on the NC tachyon for a positive $\hat{\alpha}_{px}$ corresponding to the case (iv), of our interest, the square root part of the NC action (3.20) provides additional imaginary number as follows

$$\sqrt{\hat{\beta}_p - \hat{\alpha}_{px}\hat{T}'^2} = i\sqrt{|\hat{\alpha}_{px}|\hat{T}'^2 - \hat{\beta}_p}\,, \tag{3.34}$$

and conclusively the NC action proportional to $\hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p - \hat{\alpha}_{px}\hat{T}'^2}$ becomes real. Other physical quantities, e.g., the NC energy-momentum tensor (3.26), are also real. Up to the present point, there is no reason to neglect the topological NC kink solution (iv).

From now on, let us take into account the $1/\cosh$ NC tachyon potential (3.16) and study detailed properties of the NC kinks, including computation of tension, and their various limits. In subsections 3.1–3.3, we consider the case of $\hat{U}_p(0) < 0$. As given in the upper half of table 1, there exist three types of NC kink solutions of the equation of motion (3.23). Then, in subsections 3.4–3.5, we consider the case of $\hat{U}_p(0) > 0$. As given in the lower half of table 1, there exist three more types of extended objects from the equation of motion (3.23). These three types of solutions cannot be obtained in the case $p = 1$ since the signature of $\hat{U}_p$ cannot be flipped due to negativity of $\hat{\alpha}_{1x}$ for the unstable D1-brane [20].

3.1 Array of kink and antikink for $\hat{U}_p(0) < \hat{\mathcal{E}}_p < 0$

When $\hat{U}_p(0) < \hat{\mathcal{E}}_p < 0$ ($\hat{\mathcal{T}}_p^2 > 0$ & $\hat{\alpha}_{px} < 0$), we find an exact solution of eq. (3.23)

$$\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\sqrt{\hat{u}^2 - 1}\,\sin\left(\frac{x}{\hat{\zeta}}\right), \tag{3.35}$$

where

$$\hat{u}^2 = \left|\frac{\hat{\mathcal{T}}_p^2\hat{\alpha}_{px}^2}{\hat{\beta}_p^2(\hat{T}^{11})^2}\right|, \qquad \hat{\zeta} = R\sqrt{\left|\frac{\hat{\alpha}_{px}}{\hat{\beta}_p}\right|}. \tag{3.36}$$

The NC tachyon field $\hat{T}(x)$ oscillates spatially between $\hat{T}_{\max} = R\cosh^{-1}\hat{u}$ and $-\hat{T}_{\max}$ with finite period $2\pi\hat{\zeta}$. Then the obtained configuration (3.35) is interpreted as an array of NC kink-antikink. If we consider half period of the configuration, which comprises single kink or single antikink, NC energy density of the kink (antikink) provides decent relation of the codimension-one object

$$\frac{\hat{H}}{\int dy_1\cdots dy_{p-1}} \equiv -\int_{-\frac{\pi}{2}\hat{\zeta}}^{\frac{\pi}{2}\hat{\zeta}} dx\,\sqrt{-G}\,\hat{T}^0{}_0 \tag{3.37}$$

$$= \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\int_{-\frac{\pi}{2}\hat{\zeta}}^{\frac{\pi}{2}\hat{\zeta}} dx\frac{1}{1 + (\hat{u}^2 - 1)\sin^2(x/\hat{\zeta})}$$

$$= \pi R\hat{\mathcal{T}}_p\sqrt{-\hat{\alpha}_{px}} = \pi R\mathcal{T}_p\sqrt{-\alpha_{px}}\,, \tag{3.38}$$

where we used the formula of NC energy-momentum tensor (3.27) for $\hat{T}^0{}_0$ in eq. (3.37), and eq. (3.14) and eqs. (3.31)–(3.32) for the last line (3.38). Since the factor $\sqrt{-\alpha_{px}}$ in eq. (3.38) also appeared when F1 is confined on D$(p-1)$-brane in ordinary EFT [18]

$$\mathcal{T}_{p-1} = \pi R \mathcal{T}_p \sqrt{-\alpha_{px}}\,, \tag{3.39}$$

we arrive at a conclusion that $\hat{H}/\int dy_1 \cdots dy_{p-1}$ is nothing but the tension of a unit NC kink, i.e., $\hat{H}/\int dy_1 \cdots dy_{p-1} = \mathcal{T}_{p-1} = (-G)^{(1/4)}\hat{\mathcal{T}}_{p-1}$.

The commutative limit $\theta^{\mu\nu} \to 0$ corresponds to vanishing $B_{\mu\nu}$ limit (3.12) and then smoothly continues to Minkowski spacetime (3.11). Then $-\hat{\alpha}_{px} = \hat{\beta}_p = -\alpha_{px} = \beta_p = 1$ and the decent relation without DBI type electromagnetic field is reproduced.

In the limit $\hat{T}^{11} \to 0^-$, boundary value of the tachyon field $T_{\rm max}$ approaches rapidly a true vacuum at infinity, $\lim_{T_{\rm max}\to\infty} V(T_{\rm max}) \to 0$, with keeping the half period $\pi\hat{\zeta}$ fixed, independent of the value of $\hat{T}^{11}$. Therefore, each kink (antikink) becomes a topological kink (antikink), and its NC energy density $-\sqrt{-G}\,\hat{T}^0{}_0$ is sharply peaked at each localized point of a kink (antikink). The NC energy density profile for the configuration (3.35) is expressed by a sum of δ-functions

$$\lim_{\hat{T}^{11}\to 0^-} \hat{\mathcal{H}}_p = \lim_{\hat{T}^{11}\to 0^-} -\sqrt{-G}\,\hat{T}^0{}_0 = \pi R \hat{\mathcal{T}}_p \sqrt{-\hat{\alpha}_{px}} \sum_{n=-\infty}^{\infty} \delta\left(x - n\pi\zeta\right), \tag{3.40}$$

so that each topological kink (antikink) in the array is a BPS-like object [19]. This singular limit of unit kink (antikink) with topological and BPS nature can universally be achieved for every tachyon potential satisfying eq. (3.3) [16].

In case of the array of kink and antikink, BCFT solution is already known in the presence of constant electric field [8]. Here let us compare the BCFT result and the NCFT solution. By inserting the solution (3.35) into the energy-momentum tensor (3.29) and the string current density (3.30), we obtain nonvanishing components of them

$$\begin{aligned} T_{00} &= \frac{\mathcal{T}_p \hat{V}}{\sqrt{G_0 + \hat{T}'^2}} \left(1 + \hat{T}'^2\right) \\ &= \frac{\mathcal{T}_p E_0^2}{\sqrt{G_0}} \cos^2(\pi\tilde{\lambda}) + \mathcal{T}_p \sqrt{G_0}\, \tilde{f}\left(\frac{\sqrt{G_0}x}{R}\right), \end{aligned} \tag{3.41}$$

$$T_{11} = -\frac{\mathcal{T}_p \hat{V}}{\sqrt{G_0 + \hat{T}'^2}} = -\frac{\mathcal{T}_p}{\sqrt{G_0}} \cos^2(\pi\tilde{\lambda})\,, \tag{3.42}$$

$$T_{ab} = -\mathcal{T}_p \hat{V} \sqrt{G_0 + \hat{T}'^2}\, \delta_{ab} = -\mathcal{T}_p \sqrt{G_0}\, \tilde{f}\left(\frac{\sqrt{G_0}x}{R}\right) \delta_{ab}\,, \tag{3.43}$$

$$J^{01} = \frac{\mathcal{T}_p \hat{V} E_0}{\sqrt{G_0 + \hat{T}'^2}} = \frac{\mathcal{T}_p E_0}{\sqrt{G_0}} \cos^2(\pi\tilde{\lambda})\,, \tag{3.44}$$

where $\tilde{\lambda}$ is a parameter labelling the initial pressure T_{11}, $\tilde{f}(x)$ is given by

$$\tilde{f}(\sqrt{G_0}x/R) = \frac{\cos^2(\pi\tilde{\lambda})}{\cos^4(\pi\tilde{\lambda}) + \left[1 + \cos^2(\pi\tilde{\lambda})\right] \sin^2(\pi\tilde{\lambda}) \sin^2\left(\sqrt{G_0}x/R\right)}\,, \tag{3.45}$$

and nonvanishing components of the open string metric $G_{\mu\nu}$ and noncommutative parameter $\theta^{\mu\nu}$ in this background are given by

$$-G_{00} = G_{11} = 1 - E_0^2 \overset{\text{set}}{=} G_0\,, \qquad G_{ab} = \delta_{ab} \quad (a,b = 2,3,\cdots,p)\,, \tag{3.46}$$
$$\theta^{01} = -\theta^{10} = \frac{E_0}{1-E_0^2} \overset{\text{set}}{=} \theta\,. \tag{3.47}$$

Since the BCFT results obtained through double Wick rotation [8] are for superstring theory

$$T_{00}^{\text{BCFT}} = \frac{\mathcal{T}_p E_0^2}{\sqrt{G_0}} \cos^2(\pi\tilde{\lambda}) + \mathcal{T}_p \sqrt{G_0}\, f(\sqrt{G_0}\, x)\,, \tag{3.48}$$
$$T_{11}^{\text{BCFT}} = -\frac{\mathcal{T}_p}{\sqrt{G_0}} \cos^2(\pi\tilde{\lambda})\,, \tag{3.49}$$
$$T_{ab}^{\text{BCFT}} = -\mathcal{T}_p \sqrt{G_0}\, f(\sqrt{G_0}\, x)\delta_{ab}\,, \tag{3.50}$$
$$J_{\text{BCFT}}^{01} = -T_{11}^{\text{BCFT}}\, E_0\,, \tag{3.51}$$

where

$$f(\sqrt{G_0}\, x) = \frac{\left[1+\sin^2(\pi\tilde{\lambda})\right]\cos^2(\pi\tilde{\lambda})}{\cos^4(\pi\tilde{\lambda}) + 4\sin^2(\pi\tilde{\lambda})\sin^2(\sqrt{G_0}\, x/\sqrt{2})}\,. \tag{3.52}$$

When we identify the pressure component along the transverse direction $T_{11} = T_{11}^{\text{BCFT}}$ in eq. (3.42) and eq. (3.49) with $R = \sqrt{2}$, the string charge densities coincide exactly as in eq. (3.44) and (3.51). On the other hand, the energy densities (3.41)–(3.48) and the other nonvanishing pressure components (3.43)–(3.50) are qualitatively agreed but do not match exactly due to difference between $\tilde{f}$ (3.45) and f (3.52). For bosonic string theory this kind of relations also holds.

3.2 Topological BPS NC kink for $\hat{U}_p(0) < 0$ and $\hat{\mathcal{E}}_p = 0$

When $\hat{\mathcal{E}}_p = 0$ ($\hat{\mathcal{T}}_p^2 > 0$ & $\hat{\alpha}_{px} < 0$), i.e., $\hat{\beta}_p = 0$, we find a regular monotonic solution with boundary conditions $T(\pm\infty) = \pm(\text{or } \mp)\infty$

$$\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\frac{\hat{u}}{\hat{\zeta}}x = \pm\left|\frac{\hat{\mathcal{T}}_p\sqrt{-\hat{\alpha}_{px}}}{R\hat{T}^{11}\sqrt{\hat{\beta}_p}}\right| x\,, \tag{3.53}$$

and it is single topological kink for $(\pm)$-signature (antikink for $(\mp)$-signature). Here we must note that the slope $|\frac{\hat{\mathcal{T}}_p\sqrt{-\hat{\alpha}_{px}}}{R\hat{T}^{11}\sqrt{\hat{\beta}_p}}|$ in the eq. (3.53) is finite in the limit $\hat{\beta}_p \to 0$ because 11-component of NC energy-momentum tensor is expressed by

$$\hat{T}^{11} = \frac{\hat{\mathcal{T}}_p \hat{V}\hat{\alpha}_p}{\sqrt{\hat{\beta}_p - \hat{\alpha}_{px}T'^2}\sqrt{\hat{\beta}_p}}\,.$$

The obtained solution (3.53) can also be reproduced from infinite period limit, $\lim_{\hat{\beta}_p\to 0} 2\pi\hat{\zeta} \to \infty$, at each kink site of the array (3.35), i.e., $\sin(x/\hat{\zeta}) \sim (-1)^{n+1}[(x/\hat{\zeta}) - n\pi]$ for a given n.

Since $\hat{\beta}_p = 0$, the action (3.20) divided by negative time integral and transverse volume is easily computed

$$\begin{aligned}\frac{\hat{S}}{-\int dt \int dy_1 \cdots dy_{p-1}} &= \pm \hat{\mathcal{T}}_p \sqrt{-\hat{\alpha}_{px}} \int_{-\infty}^{\infty} dx\, \hat{V}\left(\frac{d\hat{T}}{dx}\right) \\ &= \hat{\mathcal{T}}_p \sqrt{-\hat{\alpha}_{px}} \int_{-\infty}^{\infty} d\hat{T}\, \hat{V}(\hat{T}) \qquad (3.54)\\ &= \pi R \hat{\mathcal{T}}_p \sqrt{-\hat{\alpha}_{px}}\,, \qquad (3.55)\end{aligned}$$

where $+(-)$ in the first line corresponds to the kink (antikink), and the specific potential (3.8) and the solution (3.53) were used in the third line (3.55). As expected, the same decent relation (3.39) is read and the obtained topological NC kink (antikink) is interpreted as a D$(p-1)$-brane of tension satisfying $\mathcal{T}_{p-1} = \pi R \mathcal{T}_p \sqrt{-\alpha_{px}}$ as for each kink (antikink) in the previous subsection. To confirm let us compute its NC energy per unit transverse volume

$$\begin{aligned}\frac{\hat{H}}{\int dy_1 \cdots dy_{p-1}} &= \int_{-\infty}^{\infty} dx \sqrt{-G}(-\hat{T}^0{}_0) \\ &= \hat{\mathcal{T}}_p \left|\frac{\hat{\mathcal{T}}_p \hat{\alpha}_{px}}{\hat{T}^{11}}\right| \int_{-\infty}^{\infty} dx \frac{1}{1 + \left|\frac{\hat{\mathcal{T}}_p^2 \hat{\alpha}_{px}}{R^2 (T^{11})^2 \hat{\beta}_p}\right| x^2} \qquad (3.56)\\ &= \pi R \hat{\mathcal{T}}_p \sqrt{-\hat{\alpha}_{px}} = \mathcal{T}_{p-1}\,. \qquad (3.57)\end{aligned}$$

Though existence of the thick topological NC kink is guaranteed for any runaway tachyon potential (3.3), various special features involving the exact solution (3.53), the exact tension formula (3.54)–(3.55), lorentzian distribution of NC energy density (3.56), tension as NC energy per unit transverse volume (3.57), are collection of evidences showing BPS nature of the topological NC kink with non-zero thickness under the specific tachyon potential (3.8). Even though $\hat{\mathcal{T}}_p$ and $\hat{T}^{11}$ are remained to be finite, $\mathcal{T}_p$ (3.14) and T^{11} (3.33) are vanishing due to $\hat{\beta}_p = 0$. This limit corresponds to zero thickness limit with keeping BPS nature. Therefore thick NC kink requires infinite $\hat{T}^{11}$ with keeping $T^{11} = \sqrt{\hat{\beta}_p}\, \hat{T}^{11}$ finite. Since commutative limit in NCFT corresponds to no DBI type $B_{\mu\nu}$ in EFT where no single topological kink of this subsection exists except that with zero thickness in EFT [5, 7], the thick topological NC kink cannot survive in the commutative limit.

Comparison between the NCFT results and the BCFT results can easily be made for this single topological NC kink by expanding the sine function in both $\tilde{f}$ (3.45) and f (3.52) and keeping the leading term proportional to x, i.e., $\sin^2(\sqrt{G_0}\, x/\sqrt{2}) \approx G_0 x^2/2$. Then the same discussion for the array of NC kink and NC antikink is still sustained for the single topological NC kink.

3.3 Topological NC kink for $\hat{U}_p(0) < 0$ and $\hat{\mathcal{E}}_p > 0$

When $\hat{\mathcal{E}}_p > 0$ ($\hat{\mathcal{T}}_p^2 < 0$ & $\hat{\alpha}_{px} > 0$), we have a solution

$$\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\sqrt{1+\hat{u}^2}\, \sinh\left(\frac{x}{\hat{\zeta}}\right). \qquad (3.58)$$

The obtained configuration is also a topological kink (antikink) with a finite asymptotic slope $\hat{T}'(\pm\infty) = \pm R/\hat{\zeta}$. For this solution the action (3.20) per unit time and transverse volume is expressed by

$$
\begin{aligned}
\frac{\hat{S}}{-\int dt \int dy_1 \cdots dy_{p-1}} &= \hat{\mathcal{T}}_p \sqrt{\hat{\beta}_p(-\hat{u}^2)} \int_{-\infty}^{\infty} dx\, \hat{V}^2(\hat{T}(x)) \\
&= \hat{\mathcal{T}}_p \sqrt{\hat{\beta}_p(-\hat{u}^2)} \int_{-\infty}^{\infty} dx \frac{1}{1+(1+\hat{u}^2)\sinh^2(x/\hat{\zeta})} \\
&= 2i\hat{\mathcal{T}}_p R \sqrt{\hat{\alpha}_{px}} \tan^{-1} \hat{u}\,. \qquad (3.59)
\end{aligned}
$$

Since the quantity $\hat{\mathcal{T}}_p$ is imaginary as explained in eq. (3.34), the resulting action (3.59) or equivalently tension of D$(p-1)$-brane is real. As explained in eq. (3.34), the quantity $\hat{\mathcal{T}}_p$ is imaginary but the resulting action (3.59) is real and is chosen to be negative, i.e., $\hat{\mathcal{T}}_p = -i|\hat{\mathcal{T}}_p|$. So does NC energy density

$$
-\sqrt{-G}\,\hat{T}^0{}_0 = i\hat{\mathcal{T}}_p \sqrt{\hat{\beta}_p \hat{u}^2}\, \hat{V}^2(\hat{T}(x)) = \frac{i\hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}}{1+(1+\hat{u}^2)\sinh^2(x/\hat{\zeta})}\,. \qquad (3.60)
$$

It is localized near the origin and then thin limit ($\hat{T}^{11} \to 0^-$) of it can smoothly be taken.

3.4 NC bounce for $0 < \hat{\mathcal{E}}_p < \hat{U}_p(0)$

When $0 < \hat{U}_p(0)$, $\hat{\alpha}_{px} > 0$ and then negative action per unit time and unit transverse volume is read from eq. (3.21)

$$
\frac{\hat{S}}{-\int dt \int dy_1 \cdots dy_{p-1}} = \hat{\mathcal{T}}_p \sqrt{\hat{\alpha}_{px}} \int d\tilde{x}\, \hat{V}(\hat{T}) \sqrt{1 - \left(\frac{d\hat{T}}{d\tilde{x}}\right)^2}\,, \qquad (3.61)
$$

where $\tilde{x} \equiv \sqrt{\hat{\beta}_p/\hat{\alpha}_{px}}\, x$. The form of action (3.61) of static NC kinks coincides exactly with that of the rolling tachyon with or without DBI type electromagnetic coupling under exchange of time and spatial coordinates [29, 32] and, simultaneously with that of kinks corresponding to composite of D0F1 in the EFT [7, 18]. Thus, there exists a one-to-one correspondence between a rolling tachyon solution of time evolution in EFT and a kink solution with spatial distribution for $0 < \hat{U}_p(0)$ in NCFT. With this identification, the pressure $-\hat{\mathcal{T}}^{11}$ in our system corresponds to the hamiltonian density $\mathcal{H}$ in the rolling tachyon system. To be specific, let us perform detailed analysis in subsections 3.4–3.6 distinguished by the value of $\hat{\mathcal{E}}_p$.

When $0 < \hat{\mathcal{E}}_p < \hat{U}_p(0)$ ($\hat{\mathcal{T}}_p^2 > 0$, & $\hat{\alpha}_{px} > 0$), we have

$$
\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\sqrt{\hat{u}^2 - 1}\cosh\left(\frac{x}{\hat{\zeta}}\right) \qquad (3.62)
$$

which is convex up (convex down) with minimum (maximum) value $\hat{T}_{\rm min} = R\cosh^{-1}\hat{u}$ ($-\hat{T}_{\rm min}$) and has finite asymptotic slope at infinity $\hat{T}'(\pm\infty) = \pm R/\hat{\zeta}$. Therefore, this solution describes an NC bounce.

Tension per unit transverse volume of the NC bounce is computed from the action (3.61) or the NC energy

$$\begin{aligned}
\hat{\mathcal{T}}_{p-1} &= \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\int_{-\infty}^{\infty} dx\,\hat{V}^2(\hat{T}(x))\\
&= \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\int_{-\infty}^{\infty} dx\frac{1}{1+(\hat{u}^2-1)\cosh^2(x/\hat{\zeta})}\\
&= 2R\hat{\mathcal{T}}_p\sqrt{\hat{\alpha}_{px}}\tanh^{-1}\left(\frac{1}{\hat{u}}\right) = 2R\mathcal{T}_p\sqrt{\alpha_{px}}\tanh^{-1}\left(\frac{1}{u}\right) = \mathcal{T}_{p-1}\,, \qquad (3.63)
\end{aligned}$$

where we used eq. (3.14), eqs. (3.31)–(3.33), and the results of EFT in the last line.

Thin limit is achieved by taking vanishing pressure, $-\hat{T}^{11}\to 0$. Then $\hat{u}\to\infty$ and the solution (3.62) becomes singular $\hat{T}(x) \overset{-\hat{T}^{11}\to 0}{\longrightarrow} \pm\infty\cosh(|\hat{\omega}|x)$ in this limit, however this thin bounce exists in the tensionless limit ($\mathcal{T}_{p-1}\propto\tanh^{-1}(1/\infty)\to 0$) as shown in eq. (3.63).

3.5 Tensionless NC half-kink for $0<\hat{U}_p(0)$ and $\hat{\mathcal{E}}_p=\hat{U}_p(0)$

When $\hat{\mathcal{E}}_p = \hat{U}_p(0)$ ($\hat{\mathcal{T}}_p^2>0$ & $\hat{\alpha}_{px}>0$), we have a trivial ontop solution $\hat{T}(0)=0$ and nontrivial NC half-kink solution connecting smoothly unstable symmetric vacuum $\hat{T}(-\infty)=0$ and one of two stable broken vacua $\hat{T}(\infty)=\pm\infty$

$$\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\exp\left(\frac{x-x_0}{\hat{\zeta}}\right), \qquad (3.64)$$

where x_0 stands for location of the NC half-kink. Note that the scale factor $\hat{\zeta}$ (3.36) is fixed for a given R and the solution (3.64) does not have any free parameter for the given background so that thin limit of the NC half-kink cannot be taken.

If we naively compute tension of the NC half-kink from the NC energy density, it includes vacuum NC energy from the unstable vacuum, proportional to $\int_{x_0}^{\infty}dx\,\hat{V}^2(0)$. Therefore, it is reasonable to subtract the vacuum NC energy when its tension is computed. The formula for tension is obtained by inserting the NC half-kink solution (3.64) as given in the following:

$$\begin{aligned}
&\frac{\hat{S}}{-\int dt\int dy_1\cdots dy_{p-1}} - \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\int_{x_0}^{\infty}dx\,\hat{V}^2(0)\\
&= \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\left\{\int_{x_0}^{\infty}dx\left[\frac{1}{1+e^{2(x-x_0)/\hat{\zeta}}}-1\right]+\int_{-\infty}^{x_0}dx\frac{1}{1+e^{2(x-x_0)/\hat{\zeta}}}\right\}\\
&= 0\,. \qquad (3.65)
\end{aligned}$$

Amazing enough, the NC half-kink is identified as a candidate of tensionless half-brane (D$\frac{1}{2}$-brane) with thickness due to nonvanishing pressure, $-\hat{T}^{11}$. Thus, by an arbitrarily small perturbation, the zero mode x_0 may start to move to the true vacuum by transferring vacuum energy of the unstable vacuum to its kinetic energy.

3.6 Hybrid of two NC half-kinks for $0 < \hat{U}_p(0) < \hat{\mathcal{E}}_p$

When $0 < \hat{U}_p(0) < \hat{\mathcal{E}}_p$ ($\hat{\mathcal{T}}_p^2 > 0$ & $\hat{\alpha}_{px} > 0$), we have a monotonically increasing (decreasing) solution from $\hat{T}(-\infty) = -\infty$ $(+\infty)$ to $\hat{T}(\infty) = +\infty$ $(-\infty)$

$$\sinh\left(\frac{\hat{T}(x)}{R}\right) = \pm\sqrt{1-\hat{u}^2}\,\sinh\left(\frac{x}{\hat{\zeta}}\right). \tag{3.66}$$

This configuration can be regarded as a hybrid of two half-kinks joined at the origin.

Since reality condition of the solution (3.66) does not allow the limit of $\hat{u} \to \infty$, thin limit cannot be taken despite of its localized NC energy density

$$-\sqrt{-G}\hat{T}^0{}_0 = \hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\,\frac{1}{1+(1-\hat{u}^2)\sinh^2(x/\hat{\zeta})}. \tag{3.67}$$

Integration of the NC energy density (3.67) gives tension of the hybrid of two D$\frac{1}{2}$-branes

$$\hat{\mathcal{T}}_p\sqrt{\hat{\beta}_p\hat{u}^2}\int_{-\infty}^{\infty} dx\,\hat{V}^2(\hat{T}(x)) = 2R\hat{\mathcal{T}}_p\sqrt{\hat{\alpha}_{px}}\,\tanh^{-1}\hat{u} = 2R\mathcal{T}_p\sqrt{\alpha_{px}}\,\tanh^{-1}u\,. \tag{3.68}$$

We obtained three types of NC kinks for $p \geq 2$ through subsections 3.4–3.6, which are closely related with those obtained in EFT of a real tachyon coupled to U(1) gauge field [7, 18]. As those solutions are not supported in EFT without DBI type electromagnetism, the obtained solutions do not exist in commutative limit ($\lim_{B_{\mu\nu}\to 0}\theta^{\mu\nu} \to 0$).

Various other actions of the NC tachyons have been proposed [10, 11, 12], so that comparison between the NC kink solutions obtained in this paper and those from the other NC tachyon actions deserves to be analyzed but the previous discussion in ref. [20] seems enough for our cases.

4. NC tachyon kinks with NC U(1) gauge field

In the previous section we obtained all possible regular static NC kink configurations representing flat D$(p-1)$-branes from an unstable flat Dp-brane in the background of constant NS-NS two-form field. Naturally such unstable Dp-brane couples to DBI type U(1) gauge field so that, in EFT, $B_{\mu\nu}$ is replaced by $B_{\mu\nu} + F_{\mu\nu}$ which is invariant under gauge transformation of NS-NS two form field on the brane. Therefore, as far as the field strength $F_{\mu\nu}$ is constant, nothing is changed for spectrum of the NC kinks under an identification of $B_{\mu\nu} + F_{\mu\nu}$ as the gauge invariant two-form field. In fact, even if $B_{\mu\nu}$ is constant, the field strength $F_{\mu\nu}$ needs not to be constant but has complicated functional dependence of the coordinates in EFT. In the context of NCFT, the closed string metric $\eta_{\mu\nu}$ and NS-NS two-form field $B_{\mu\nu}$ are replaced by the open string metric $G_{\mu\nu}$ (3.11) and NC parameter $\theta^{\mu\nu}$ (3.12), and then the field strength $F_{\mu\nu}$ is given by the NC field strength $\hat{F}_{\mu\nu}$. In this section let us include the NC gauge field $\hat{A}_\mu$ resulting in

$$\hat{F}_{\mu\nu}(x) = \partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]_*\,, \tag{4.1}$$

where single x-dependence is consistent with flatness of codimension-one D-brane and the assumption on the NC tachyon field (3.18). Here the NC commutator in eq. (4.1) is defined by

$$[A,B]_* = A*B - B*A\,. \tag{4.2}$$

Let us begin with the following action [20] similar to eq. (3.13)

$$\hat{S}_* = -\frac{\hat{\mathcal{T}}_p}{2}\int d^{p+1}x\,\left[\hat{V}(\hat{T})*\sqrt{-\hat{X}_*}+\sqrt{-\hat{X}_*}*\hat{V}(\hat{T})\right], \tag{4.3}$$

where the determinant $\hat{X}_{\hat{F}}$ involves $\hat{F}_{\mu\nu}$ as

$$\hat{X}_* \equiv \det{}_*\left[G_{\mu\nu}+\hat{F}_{\mu\nu}+\frac{1}{2}\left(\hat{D}_\mu\hat{T}*\hat{D}_\nu\hat{T}+\hat{D}_\nu\hat{T}*\hat{D}_\mu\hat{T}\right)\right], \tag{4.4}$$

where NC covariant derivative is

$$\hat{D}_\mu = \partial_\mu - i[\hat{A}_\mu,\,]_*\,. \tag{4.5}$$

When the tachyon effective action (3.1) was derived [31] and the equivalence between DBI action and NCFT action was verified [9], the condition of slowly varying fields was assumed for both the tachyon field as $\hat{\partial}_\mu\hat{D}_\nu\hat{T}$ and the gauge field as $\hat{\partial}_\mu\hat{F}_{\nu\rho}=0$. Therefore, we can employ a rather simple form of the NC action [20] connected smoothly to the DBI type NC action by Seiberg-Witten [9]

$$\hat{S}_{\hat{F}} = -\hat{\mathcal{T}}_p\int d^{p+1}x\,\left[\hat{V}(\hat{T})\sqrt{-\hat{X}_{\hat{F}}}+\mathcal{O}(\partial\hat{F},\,\partial\hat{D}\hat{T})\right], \tag{4.6}$$

where

$$\hat{X}_{\hat{F}} = \det(G_{\mu\nu}+\hat{F}_{\mu\nu}+\hat{D}_\mu\hat{T}\hat{D}_\nu\hat{T})\,. \tag{4.7}$$

In addition we will show that the kink configurations from eq. (4.6) are exactly the same as those from eq. (4.3), which is likely to support a kind of universality.

NC tachyon equation and NC gauge equation from the action (4.6) are

$$D_\mu\left(\frac{\hat{V}(\hat{T})}{\sqrt{-\hat{X}}}\,\hat{C}_{\rm S}^{\mu\nu}\hat{D}_\nu\hat{T}\right)-\left[\frac{\hat{T}}{R^2}\frac{d\hat{V}(\hat{T})}{d\hat{T}}\right]_*\sqrt{-\hat{X}}=0\,, \tag{4.8}$$

$$\hat{D}_\mu\left(\frac{\hat{V}(\hat{T})}{\sqrt{-\hat{X}_{\hat{F}}}}\,\hat{C}_{\rm A}^{\mu\nu}\right)+i\left[\hat{T}\,,\,\frac{\hat{V}(\hat{T})}{\sqrt{-\hat{X}_{\hat{F}}}}\,\hat{C}_{\rm S}^{\mu\nu}\hat{D}_\nu\hat{T}\right]_*=0\,, \tag{4.9}$$

where $\hat{C}_{\rm S}^{\mu\nu}$ and $\hat{C}_{\rm A}^{\mu\nu}$ are symmetric and antisymmetric parts of cofactors, respectively. Or some of the equations of motion can be replaced by conservation of NC energy-momentum [33]

$$\hat{D}_\mu\hat{T}^{\mu\nu}=0 \tag{4.10}$$

for calculational simplicity.

Since we assumed x-dependence to both the NC tachyon field (3.18) and the NC field strength tensor (4.1), difficulty of solving the equations of motion (4.8)–(4.9) with Bianchi identity for the NC gauge field [33]

$$\hat{D}_\mu \hat{F}_{\nu\rho} + \hat{D}_\nu \hat{F}_{\rho\mu} + \hat{D}_\rho \hat{F}_{\mu\nu} = 0 \tag{4.11}$$

is significantly reduced. The simplest case is $p = 1$ where the Bianchi identity (4.11) becomes trivial, so analysis was made and results were obtained, which are consistent with those of EFT [20]. From now on let us tackle the case of D2-brane ($p = 2$) on which dynamics of the U(1) gauge field is expected to be much more complicated due to three components of electromagnetic field strength tensor and three NC parameters.

For a flat D2-brane, the closed string metric is $g_{\mu\nu} = \eta_{\mu\nu}$ and we set components of the NS-NS two-form field $B_{\mu\nu}$ as

$$B_{01} = E_1\,, \qquad B_{02} = E_2\,, \qquad B_{12} = B\,, \qquad \text{with} \qquad \mathbf{E}^2 = E_1^2 + E_2^2\,, \tag{4.12}$$

and then the open string metric $G_{\mu\nu}$ and the NC parameter $\theta^{\mu\nu}$ are determined by eq. (3.11) and eq. (3.12), respectively;

$$\begin{aligned} G_{00} &= -(1-\mathbf{E}^2)\,, \qquad G_{01} = E_2 B\,, \qquad G_{02} = -E_1 B\,, \\ G_{11} &= 1 - E_1^2 + B^2\,, \qquad G_{12} = -E_1 E_2\,, \qquad G_{22} = 1 - E_2^2 + B^2\,, \end{aligned} \tag{4.13}$$

and

$$\theta^{01} = \frac{E_1}{1-\mathbf{E}^2+B^2}\,, \qquad \theta^{02} = \frac{E_2}{1-\mathbf{E}^2+B^2}\,, \qquad \theta^{12} = \frac{-B}{1-\mathbf{E}^2+B^2}\,. \tag{4.14}$$

From the relation (3.14) we have

$$G_{\rm s} = (-G)^{1/4} g_{\rm s} = \sqrt{1-\mathbf{E}^2+B^2}\, g_{\rm s}\,, \qquad \hat{\mathcal{T}}_2 = (-G)^{-1/4}\mathcal{T}_2 = \frac{\mathcal{T}_2}{\sqrt{1-\mathbf{E}^2+B^2}}\,. \tag{4.15}$$

Since we are interested in obtaining flat D1-branes or D1F1 composites from the flat unstable D2-brane, all the NC fields of our consideration depend on x-coordinate but do not depend on y-coordinate as given in eq. (3.18) and eq. (4.1). Suppose we choose a noncovariant gauge for the NC U(1) gauge field as

$$\theta^{1\mu}\hat{A}_\mu = 0\,, \tag{4.16}$$

where the time component $\hat{A}_0$ is expressed in terms of $\hat{A}_2$. Inserting this into the NC field strength tensor $\hat{F}_{\mu\nu}$, we obtain

$$\hat{F}_{01} \equiv \hat{E}_1 = \partial_0\hat{A}_1 - \partial_1\hat{A}_0 - i[\hat{A}_0,\hat{A}_1]_* = \partial_0\hat{A}_1 + \frac{\theta^{12}}{\theta^{10}}\partial_1\hat{A}_2 + i\frac{\theta^{12}}{\theta^{10}}[\hat{A}_2,\hat{A}_1]_*\,, \tag{4.17}$$

$$\hat{F}_{02} \equiv \hat{E}_2 = \partial_0\hat{A}_2 - \partial_2\hat{A}_0 - i[\hat{A}_0,\hat{A}_2]_* = \partial_0\hat{A}_2 + \frac{\theta^{12}}{\theta^{10}}\partial_2\hat{A}_2\,, \tag{4.18}$$

$$\hat{F}_{12} \equiv \hat{B} = \partial_1\hat{A}_2 - \partial_2\hat{A}_1 - i[\hat{A}_1,\hat{A}_2]_*\,. \tag{4.19}$$

Eliminating the commutator term in eq. (4.17) and eq. (4.19), we have two first-order equations of $\hat{A}_1$ and $\hat{A}_2$ in t and y, i.e., they are eq. (4.18) and

$$(\theta^{10}\partial_0 + \theta^{12}\partial_2)\hat{A}_1 = \theta^{10}\hat{E}_1(x) - \theta^{12}\hat{B}(x)\,. \tag{4.20}$$

Their general solutions are

$$\hat{A}_1 = \frac{1}{\theta^{12}}\left[c_1\theta^{12}t + (1-c_1)\theta^{10}y\right]\hat{E}_1(x) - \frac{1}{\theta^{10}}\left[c_2\theta^{12}t + (1-c_2)\theta^{10}y\right]\hat{B}(x) + g_1(x)\,, \tag{4.21}$$

$$\hat{A}_2 = \frac{1}{\theta^{12}}\left[c_3\theta^{12}t + (1-c_3)\theta^{10}y\right]\hat{E}_2(x) + g_2(x)\,, \tag{4.22}$$

where c_is ($i = 1,2,3$) are arbitrary constants and $g_i(x)$s ($i = 1,2$) are arbitrary real functions of x-coordinate. Substituting $\hat{A}_1$ and $\hat{A}_2$ into eq. (4.19), we have from the terms proportional to θ^{20}

$$c_1 = c_2 = c_3 = c\,. \tag{4.23}$$

The gauge fixing condition (4.16) gives an expression of $\hat{A}_0$;

$$\begin{aligned}\hat{A}_0 &= -\frac{\theta^{12}}{\theta^{10}}\hat{A}_2 \\ &= -\frac{1}{\theta^{10}}\left[c\theta^{12}t + (1-c)\theta^{10}y\right]\hat{E}_2(x) - \frac{\theta^{12}}{\theta^{10}}g_2(x)\,.\end{aligned} \tag{4.24}$$

Substituting the NC gauge field (4.21)–(4.24) and its field strength (4.1) into the Bianchi identity (4.11), we notice disappearance of arbitrariness

$$(1 - \theta^{10}\hat{E}_1 + \theta^{12}\hat{B})\hat{E}_2' - \hat{E}_2(-\theta^{10}\hat{E}_1' + \theta^{12}\hat{B}') = 0\,, \tag{4.25}$$

which implies that c and g_i are remnants of gauge artifacts. Since eq. (4.25) results in an algebraic relation among NC parameters and NC field strength tensor

$$1 - \theta^{10}\hat{E}_1 + \theta^{12}\hat{B} = c_4\hat{E}_2\,, \tag{4.26}$$

again the NC gauge field (4.21)–(4.24) is inserted into eq. (4.19) or eq. (4.17). Then, with the help of the Bianchi identity (4.26), the arbitrary constants, c and c_4, and arbitrary functions $g_i(x)$s should satisfy a constraint equation

$$c(1 - c_4\hat{E}_2) - \theta^{10}\hat{E}_1 + \theta^{12}c_4\hat{E}_2 g_2' + \theta^{10}\theta^{12}\hat{E}_2 g_1' = 0\,. \tag{4.27}$$

Since the NC tachyon field depends only on x-coordinate as given in eq. (3.18) and covariant derivative of the NC tachyon is expressed in terms of constant (NC) parameters and $\hat{E}_2$ as

$$\hat{D}_\mu\hat{T} = \hat{E}_2\left(\delta_{\mu 0}\theta^{12} + \delta_{\mu 1}c_4 - \delta_{\mu 2}\theta^{10}\right)\hat{T}'\,, \tag{4.28}$$

every star product between two NC tachyon fields is replaced by an ordinary product

$$\hat{T} * \hat{T} = \hat{T}^2\,, \qquad \hat{D}_\mu\hat{T} * \hat{D}_\nu\hat{T} = \hat{D}_\mu\hat{T}\hat{D}_\nu\hat{T}\,. \tag{4.29}$$

Here we easily notice that a set of constant NC field strength tensor, $(\hat{E}_1, \hat{E}_2, \hat{B})$, is a solution of the NC Bianchi identity (4.25). In order to show that the constant NC field strength tensor is unique solution of the NC equations of motion (4.8)–(4.9) under the ansatz (3.18) and (4.1), let us consider the conservation of NC energy-momentum (4.10), giving

$$\hat{D}_\mu \hat{T}^{\mu\,0} = -\sqrt{-G}\hat{E}_2\hat{B}'\hat{\gamma} + \omega_0\hat{\gamma}' = 0\,, \tag{4.30}$$

$$\hat{D}_\mu \hat{T}^{\mu\,1} = \sqrt{-G}\hat{E}_2\hat{E}_2'\hat{\gamma} + \omega_1\hat{\gamma}' = 0\,, \tag{4.31}$$

$$\hat{D}_\mu \hat{T}^{\mu\,2} = -\sqrt{-G}\hat{E}_2\hat{E}_1'\hat{\gamma} + \omega_2\hat{\gamma}' = 0\,, \tag{4.32}$$

where we set

$$\hat{\gamma} = \frac{\hat{\mathcal{T}}_2\hat{V}}{\sqrt{-\hat{X}_*}}\,, \tag{4.33}$$

and ω_0, ω_1, and ω_2 are

$$\begin{aligned}
\omega_0 &= \sqrt{-G}\Big[-\hat{B} + E_1^2 B + E_2 B(1 - \mathbf{E}^2 + B^2)\,c_4 + (1 + B^2)B\Big]\hat{E}_2\,,\\
\omega_1 &= \sqrt{-G}\Big[\hat{E}_2 - E_1^2 E_2 + E_2 B^2 - (1 - E_2^2)(1 - \mathbf{E}^2 + B^2)\,c_4\Big]\hat{E}_2\,,\\
\omega_2 &= \sqrt{-G}\Big[-\hat{E}_1 - E_1 E_2(1 - \mathbf{E}^2 + B^2)\,c_4 - E_1 B^2 - (1 - E_1^2)E_1\Big]\hat{E}_2\,.
\end{aligned} \tag{4.34}$$

We multiply some coefficients to eqs. (4.30)–(4.32) and add all of them. Then we obtain

$$\begin{aligned}
0 &= -\theta^{10}\hat{D}_\mu\hat{T}^{\mu\,2} + \theta^{12}\hat{D}_\mu\hat{T}^{\mu\,0} + \frac{1 - \theta^{10}\hat{E}_1 + \theta^{12}\hat{B}}{\hat{E}_2}\hat{D}_\mu\hat{T}^{\mu\,1}\\
&= \Big[(1 - \theta^{10}\hat{E}_1 + \theta^{12}\hat{B})\hat{E}_2' - \hat{E}_2(-\theta^{10}\hat{E}_1' + \theta^{12}\hat{B}')\Big]\,\hat{\gamma} + \epsilon\hat{\gamma}'\\
&= \epsilon\hat{\gamma}'\,,
\end{aligned} \tag{4.35}$$

where ϵ is defined by

$$\begin{aligned}
\epsilon &\equiv -\theta^{10}\omega_2 + \theta^{12}\omega_0 + \frac{1 - \theta^{10}\hat{E}_1 + \theta^{12}\hat{B}}{\hat{E}_2}\omega_1\\
&= \frac{\hat{E}_2}{1 - \mathbf{E}^2 + B^2}\Big[1 - 2E_1^2 + E_1^4 - E_2^2 - 2E_1^2 B^2 - B^4 -\\
&\qquad - (1 - \mathbf{E}^2 + B^2)\,c_4\Big\{2E_1^2 E_2 - (1 - E_2^2)(1 - \mathbf{E}^2 + B^2)c_4\Big\}\Big]\,,
\end{aligned} \tag{4.36}$$

and, in the last line (4.35), we used the Bianchi identity (4.25). Only trivial solutions are provided by $\epsilon = 0$, and then equation of constant $\hat{\gamma}$ should be examined for nontrivial NC tachyon solutions. Once $\hat{\gamma}$ is constant, eqs. (4.30)–(4.32) force constant NC electromagnetic field as follows

$$\frac{\hat{E}_2\hat{B}'}{\omega_0} = -\frac{\hat{E}_1\hat{E}_2'}{\omega_1} = \frac{\hat{E}_2\hat{E}_1'}{\omega_2} = \frac{\hat{\gamma}'}{\hat{\gamma}} = 0\,. \tag{4.37}$$

Therefore, the solution of constant $\hat{\gamma}$ and constant NC electromagnetic field $\hat{F}_{\mu\nu}$ is unique solution of the Bianchi identity (4.25) and the conservation of NC energy-momentum tensor (4.30)–(4.32). If we insert this constant solution into the equations of NC tachyon (4.8) and NC gauge field (4.9), it automatically satisfies the equations.

The only nontrivial equation to be solved now is that of constant $\hat{\gamma}$ (4.33). In order to use the results of the previous section, let us compute the determinant in eq. (4.33). Though every component of $\hat{X}_{\mu\nu}$ contains the term of $\hat{T}'^2$, all the quartic terms $\hat{T}'^4$ and sixth-order terms $\hat{T}'^6$ vanish but constant and quadratic terms survive

$$\begin{aligned}
-\hat{X}_{\hat{F}} &= -\det\Big[G_{\mu\nu} + \hat{F}_{\mu\nu} + \hat{E}_2^2(\delta_{\mu 0}\theta^{12} + \delta_{\mu 1}\, c_4 - \delta_{\mu 2}\theta^{10})(\delta_{\nu 0}\theta^{12} + \delta_{\nu 1}\, c_4 - \delta_{\nu 2}\theta^{10})\hat{T}'^2\Big] \\
&= -\det(G_{\mu\nu} + \hat{F}_{\mu\nu}) - \hat{C}^{\mu\nu}_{\hat{F}}\Big|_{\hat{T}'=0} \hat{X}^{\hat{F}}_{\mu\nu}\Big|_{G_{\mu\nu}=\hat{F}_{\mu\nu}=0} \qquad (4.38) \\
&= \hat{\beta}_{\hat{F}} - \hat{\alpha}_{\hat{F}}\hat{T}'^2 \,, \qquad (4.39)
\end{aligned}$$

where $\hat{\beta}_{\hat{F}} = \det(G_{\mu\nu} + \hat{F}_{\mu\nu})$ and explicit form of $\hat{\alpha}_{\hat{F}}$ is

$$\begin{aligned}
\hat{\alpha}_{\hat{F}} = \Big[&(G_{11}G_{22} - G_{12}^2 + \hat{F}_{12}^2)(\theta^{12})^2 + (G_{00}G_{22} - G_{02}^2 + \hat{F}_{02}^2)c_4^2 + \\
&+ (G_{00}G_{11} - G_{01}^2 + \hat{F}_{01}^2)(\theta^{01})^2 - 2(G_{01}G_{22} + G_{02}G_{12} - \hat{F}_{02}\hat{F}_{12})c_4\theta^{12} - \\
&- 2(G_{01}G_{12} + G_{11}G_{02} + \hat{F}_{01}\hat{F}_{12})\theta^{01}\theta^{12} - \\
&- 2(G_{00}G_{12} - G_{01}G_{02} + \hat{F}_{01}\hat{F}_{02})\theta^{01}c_4\Big]\hat{E}_2^2 \,. \qquad (4.40)
\end{aligned}$$

For $\hat{X}^{\hat{F}}_{\mu\nu}|_{G_{\mu\nu}=\hat{F}_{\mu\nu}=0}$ in eq. (4.38), we pretended $\hat{F}_{\mu\nu} = 0$ for convenience but actually $\hat{E}_2(\equiv \hat{F}_{02}) \neq 0$. Then the equation of constant $\hat{\gamma}$ (4.33) is rewritten in a form of conservation of mechanical energy $\mathcal{E}_{\hat{F}}$ of a unit-mass particle in 1-dimensional motion, of which coordinate is $\hat{T}$, time x, and potential $U_{\hat{F}}$

$$\hat{\mathcal{E}}_{\hat{F}}(\hat{T}) = \frac{1}{2}\hat{T}'^2 + \hat{U}_{\hat{F}}(\hat{T}) \,, \qquad (4.41)$$

where

$$\hat{\mathcal{E}}_{\hat{F}}(\hat{T}) = \frac{\hat{\beta}_{\hat{F}}}{2\hat{\alpha}_{\hat{F}}} \,, \qquad \hat{U}_{\hat{F}}(\hat{T}) = \frac{1}{2\hat{\alpha}_{\hat{F}}\hat{\gamma}^2}[\hat{\mathcal{T}}_2\hat{V}(\hat{T})]^2 \,. \qquad (4.42)$$

Eq. (4.41) is formally the same first-order equation as eq. (3.23). At the beginning we had six free parameters, namely, three NC parameters $(\theta^{01}, \theta^{02}, \theta^{12})$ (or equivalently constant background NS-NS 2-form field (E_1, E_2, B)) and three constant NC electromagnetic field $(\hat{E}_1, \hat{E}_2, \hat{B})$, and an integration constant $\hat{\gamma}$, however general NC kink solutions of eq. (4.41) are classified by two parameters $\hat{\mathcal{E}}_{\hat{F}}$ and $\hat{U}_{\hat{F}}(\hat{T} = 0)$. Therefore, the analysis in the previous section is applicable exactly in the same manner. As far as we have runaway NC tachyon potential (3.3), we obtain six types of NC kink solutions and exact solutions for the $1/\cosh$-potential (3.8) as shown in table 1 and eq. (3.9). Here we do not repeat the same detailed analysis.

Since antisymmetric part of the cofactor $\hat{C}^{\mu\nu}_{\hat{F}}$ of $\hat{X}^{\hat{F}}_{\mu\nu}$ does not vanish, conjugate momentum $\hat{\Pi}^i = \hat{\gamma}\hat{C}^{0i}_{\hat{F}}$ of the NC gauge field $\hat{F}_{\mu\nu}$ also does not vanish

$$\hat{\Pi}^1 = \hat{\gamma}\hat{C}^{01}_{\hat{F}\mathrm{A}} = \hat{\gamma}(G_{20}\hat{B} - G_{21}\hat{E}_2 + G_{22}\hat{E}_1 - \theta^{01}\hat{E}_2^2\hat{T}'^2) \,, \qquad (4.43)$$

$$\hat{\Pi}^2 = \hat{\gamma}\hat{C}^{02}_{\hat{F}\mathrm{A}} = \hat{\gamma}(-G_{10}\hat{B} - G_{11}\hat{E}_2 - G_{12}\hat{E}_1 + c_4\hat{E}_2^2\hat{T}'^2) \,, \qquad (4.44)$$

where constant $\hat{\gamma}$ is given in eq. (4.33). Both $\hat{\Pi}^1$ and $\hat{\Pi}^2$ involve localized piece proportional to $\hat{T}'^2$ near the position of D1-brane $x = 0$ in addition to constant part. The constant part can be interpreted as constant F1 fluid density [34, 35, 36] and the localized piece stands for confined F1 along D1 [7, 18]. Therefore, the obtained objects are identified with D1F1 bound in the background of constantly distributed F1s, i.e., they are array of D1F1-$\bar{\mathrm{D}}$1F1, single D1F1, half-DF1, and their composites.

When the DBI type NC electromagnetic field is added, the resultant NC action (4.3) is proven to be equivalent to that of commutative EFT only up to the leading NC parameter $\mathcal{O}(\theta^{\mu\nu})$ [20]. A noteworthy observation of this section is the fact that spectrum of the NC kinks qualitatively coincides with that of commutative EFT. So it might be intriguing to study the relation between the NCFT action and the commutative EFT action further.

5. Summary and discussion

In this paper we considered kink (domain wall) solutions in various NCFTs. For the NCFT of a real scalar field in flat metric, we showed that all the kink solutions in ordinary scalar field theory with quadratic derivative term and polynomial scalar potential become NC kink solutions in NCFT. In ϕ^4-theory the obtained decent relation, given by the ratio between the local maximum of the potential and energy of the NC kink, has about 20% difference in its coefficient from that of string theory.

We studied DBI type effective action of tachyon field with constant open string metric and NC parameter, and obtained all possible static NC kink solutions classified by array of NC kink-antikink, BPS or non-BPS single topological NC kink, tensionless NC half-kink, NC bounce, and hybrid of two half-kinks. For specific $1/\cosh$ type NC tachyon potential, those are given by exact solutions and subsequently decent relation between tension of an unstable Dp-brane and stable D$(p-1)$-brane is also reproduced in the same as that of EFT and BCFT, which supports identification of the unit NC kink as a stable codimension-one D-brane. When DBI type U(1) NC gauge field is turned on on an unstable D2-brane, gauge equation and NC Bianchi identity dictate that every field strength component should be constant. Therefore, forms of the obtained solutions are the same as those without the NC gauge field, but they carry F1 charge localized on the codimension-one D-brane. Extension to the case of arbitrary p is extremely complicated but is likely to result in the same constant field strength of NC gauge field.

UV-IR mixing is a representative property in NCFT [21] and it is realized in GMS solitons [13], however it needs further study to answer the question whether or not NC tachyon kinks share such property. We obtained all possible kinks interpreted as flat codimension-one branes, and NC tachyon tubes from tachyon tubes [38] are worth being reproduced as for thin objects [37]. Since NCFT is an effective theory, it is definitely intriguing that we obtain those configurations in the context of (boundary) string field theory [39].

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